

A Novel 3D Hybrid FEM-PO Technique for the Analysis of Scattering Problems

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Abstract

A novel three-dimensional hybrid Finite Element Method (FEM) and Physical Optics (PO) technique for the efficient analysis of scattering problems is presented. It makes use of FEM for the regions with small and complex features and PO for the analysis of the electrically large objects of the structure taking into account mutual interactions between the FEM domains and the objects analyzed with PO. The hybrid method proposed makes use of an iterative FEM for open region problems that allows the FEM domain to be truncated with a minimum number of unknowns while retaining the original sparse and banded structure of the FEM matrices, and also an easy hybridization with other numerical techniques (as PO in this paper). Several numerical results are given showing some of the features of the method.

1 Introduction

In this paper, a novel three-dimensional (3D) hybrid FEM and High Frequency (HF) methodology for the efficient analysis of general scattering problems is presented. It makes use of FEM for the regions with small and complex features (arbitrary geometry and/or inhomogeneous—even anisotropic—media) and a HF technique for the analysis of the electrically large objects of the structure, taking into account mutual interactions between the FEM domains and the objects analyzed with the HF technique. The hybrid methodology proposed is independent of the particular HF technique used. However, the implementation presented in this paper corresponds to the use of Physical Optics (PO) as the HF technique. PO has been chosen because of its accuracy when the interaction of the FEM domain and the electrically large objects is in the near field and because it allows an easy implementation with distributed sources (as those provided by FEM).

The FEM part is an improved implementation of the iterative methodology for general open-region problems presented in [1], [2]. This FEM method allows an easy hybridization with other techniques. Examples of hybridization with PO in the context of scattering problems was presented in [3] and also for

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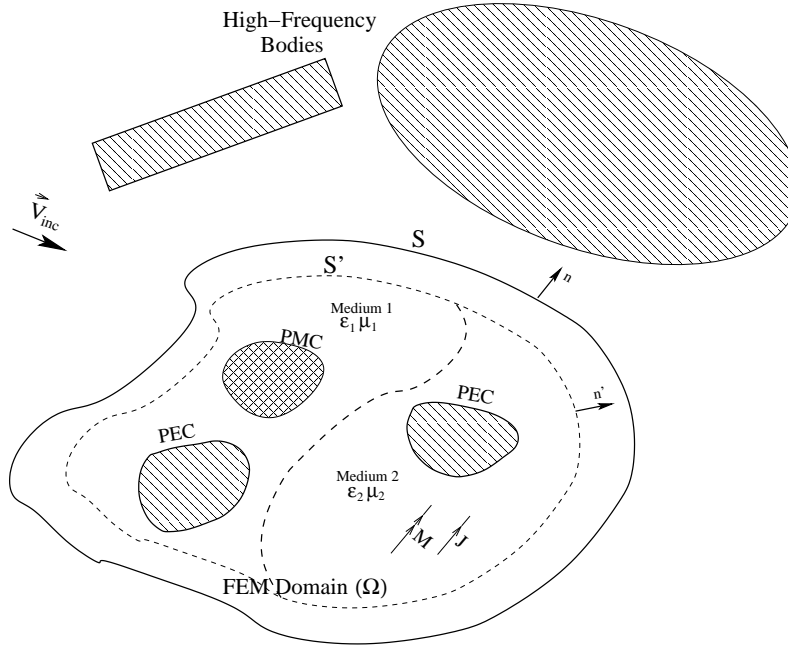


Fig. 1: Hybrid method analysis for a scattering problem

radiation problems in [4]. In both cases, the hybridization was restricted to two-dimensional problems. In this paper, a 3D implementation of this hybrid method is presented for the first time. The hybrid method is described in Section 2. Several numerical results of its application are shown in Section 3 showing some of the features of the method.

2 Hybrid Method

As the hybrid methodology proposed is independent of the particular HF technique used, the description of the method made below is general, i.e., valid for any HF method. Particularization to PO as the HF technique is made only when necessary.

Consider the problem illustrated in Fig. 1. It consists of a region (bounded by an arbitrarily shaped auxiliary boundary S') with small features and a complex configuration where there are several materials (which may be anisotropic and with electric and/or magnetic losses) and conducting objects of arbitrary shape. A rigorous method such as FEM is needed for the analysis. The region external to S' consists of an open homogeneous medium with the presence of some electrically large objects (typically conducting and dielectric coated conducting objects). The electrical size of those objects makes appropriate the use of an (approximate) asymptotic technique for the analysis of that region. Thus, these objects will be referred as HFBs (HF Bodies). An incident field is considered as excitation.

The FEM domain (Ω) is not truncated at S' but at a boundary S , located outside S' and in such a way that the region between S and S' is homogeneous. Boundary S may be arbitrarily shaped but typically it is selected conformal to S' . Distance from S' to S is usually small, typically in the range of 0.05λ to 0.2λ . Thus, the FEM domain can be truncated very close to the sources of the problem.

The FEM formulation is based on the vector wave equation,

$$\nabla \times \left(f_r^{-1} \nabla \times \vec{V} \right) - k_0^2 g_r \vec{V} = 0, \quad (1)$$

	\vec{V}	f_r	f_r	PEC	PMC
\vec{E} Formulation	\vec{E}	ε_r	μ_r	$\hat{n} \times \vec{V} = 0$	$\hat{n} \times (f_r^{-1} \nabla \times \vec{V}) = 0$
\vec{H} Formulation	\vec{H}	μ_r	ε_r	$\hat{n} \times (f_r^{-1} \nabla \times \vec{V}) = 0$	$\hat{n} \times \vec{V} = 0$

Table 1: Correspondences for FEM formulation

where k_0 is the free space number wave, \vec{V} refers to \vec{E} or \vec{H} , and f_r, g_r are defined accordingly (see Table 1). The boundary conditions inside S may be of Perfect Electric Conductor (PEC) and Perfect Magnetic Conductor (PMC) that corresponds to Dirichlet or Neumann type depending on the formulation used (see Table 1).

A Cauchy boundary condition is used on S

$$\hat{n} \times (\nabla \times \vec{V}) + j k_0 \hat{n} \times (\hat{n} \times \vec{V}) = \vec{\Psi} \quad (2)$$

Differential equation (1) is discretized inside Ω and a sparse system of equations is obtained

$$\begin{bmatrix} K_{II} & K_{IS} \\ K_{SI} & K_{SS} \end{bmatrix} \begin{Bmatrix} \{g_I\} \\ \{g_S\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{b_\Psi\} \end{Bmatrix} \quad (3)$$

where the sub-indexes S and I refer to the degrees of freedom g on S and in the interior region, respectively. Vector b_Ψ is a function of the value of the boundary condition on S , i.e., $\vec{\Psi}$ (see (2)).

In order for (3) to be solved b_Ψ , i.e., $\vec{\Psi}$, must be known. An initial value of $\vec{\Psi}$, $\vec{\Psi}^{(0)}$ is calculated as

$$\vec{\Psi}^{(0)} = \hat{n} \times (\nabla \times \vec{V}^{(0)}) + j k_0 \hat{n} \times (\hat{n} \times \vec{V}^{(0)}) \quad (4)$$

where $\vec{V}^{(0)}$ corresponds to the incident field in the presence of the HFBs, i.e., $\vec{V}^{(0)} = \vec{V}^{inc} + \vec{V}_{\text{HFT}}^{inc}$ being \vec{V}^{inc} the incident wave of Fig. 1 and $\vec{V}_{\text{HFT}}^{inc}$, the incident wave field scattered by the HFBs—in absence of FEM domain—. Then (3) is solved. From the FEM field solution and making use of the Equivalence Principle, the value of the scattered field (and its curl) on S are computed from the values of the field and its curl on the auxiliary boundary S' . The contributions of HFBs are calculated using a HF technique, i.e.,

$$\vec{V}^{sc}(\vec{r}) = \oint_{S'} \left[-(\hat{n}' \times \vec{V}(\vec{r}')) \times \nabla G + (\hat{n}' \times (\nabla \times \vec{V}(\vec{r}'))) \cdot \left(G + \frac{1}{k_0^2} \nabla \nabla G \right) \right] d\vec{s}' + \vec{V}_{\text{HFT}}^{sc} \quad (5)$$

$$\nabla \times \vec{V}^{sc}(\vec{r}) = \oint_{S'} \left[-(\hat{n}' \times (\nabla \times \vec{V}(\vec{r}'))) \times \nabla G + (\hat{n}' \times \vec{V}(\vec{r}')) \cdot (k_0^2 G + \nabla \nabla G) \right] d\vec{s}' + \nabla \times \vec{V}_{\text{HFT}}^{sc} \quad (6)$$

where $\vec{r} \in S$, $\vec{r}' \in S'$, and G corresponds (typically but not necessarily) to the free space Green's function. Specifically, in the implementation presented in this paper $\vec{V}_{\text{HFT}}^{sc}$ and $\nabla \times \vec{V}_{\text{HFT}}^{sc}$ correspond to the scattered field and its curl computed by PO. The total field is obtained as $\vec{V}^{sc} = \vec{V}^{(0)} + \vec{V}^{sc}$ and a new value of $\vec{\Psi}$, $\vec{\Psi}^{(1)}$, is obtained using (2).

In order to compute $\vec{V}_{\text{HFT}}^{sc}$ (and $\nabla \times \vec{V}_{\text{HFT}}^{sc}$), the radiated field on HFBs from the equivalent sources on S' is calculated, mutual interactions between HFBs are applied, and the reradiated field (from HFBs to

boundary S) is computed. This process, although implemented as above, is conceptually equivalent to the use of a modified Green's function $G_m(\vec{r}, \vec{r}')$ to compute the values of the scattered field \vec{V}^{sc} (and its curl) on S from the equivalent currents on S' , i.e., where $G_m(\vec{r}, \vec{r}') = G(\vec{r}, \vec{r}') + G_{\text{HFT}}(\vec{r}, \vec{r}')$. See Fig. 2.

Thus, the HF method is easily hybridized with FEM. The new value $\vec{\Psi}^{(1)}$ is compared with the previous one $\vec{\Psi}^{(0)}$. In general, $\vec{\Psi}^{(1)}$ will be different from $\vec{\Psi}^{(0)}$. Then, the FEM system (3) is solved using $\vec{\Psi}^{(1)}$ as the value of the boundary condition. The iterative process continues until a certain error criteria is satisfied; typically, the L_2 -norm of the difference between the values of $\vec{\Psi}$ between two consecutive iterations being lower than a given value.

Fig. 2 shows the flow chart of this iterative hybrid technique. The method described above has several advantages: the FEM domain may be truncated with a minimum number of unknowns, the original sparse and banded structure of the FEM matrices is retained, existing code for non-open region problems may be reutilized, and an easy implementation of the hybridization by means of a simple modification of the iterative FEM loop is obtained. All that is achieved at the expense of performing a number of iterations where the numerical cost of the second and subsequent iterations is very small. If a direct solver is used to solve the FEM problem at each iteration cycle, the factorization of the FEM matrix must be performed only once at the first iteration. Thus, the FEM solution for the second and subsequent iterations are obtained by simple backward substitution. If an iterative solver is used, the solution of the previous iteration cycle may be used as an initial guess for the next iteration of the solver.

It is worth noting that the problem of Fig. 1 depicts a simplified example used to describe the hybrid approach. In a general case, several FEM domains may exist. Also, infinite ground planes may be taken into account analytically using the free space Green's function with image theory. Numerical results are shown in the next section.

3 Results

An example of a dielectric ($\epsilon_r = 2 - j$, $\mu_r = 1$) coated metallic cube above a perfect conductor plane (see Fig. 3) has been chosen to illustrate some of the features of the hybrid method. The dielectric boundary is used as the auxiliary surface S' . The FEM domain boundary S is chosen conformal to S' . The FEM domain is discretized with an irregular mesh of 1053 tetrahedra that corresponds to 7694 degrees of freedom making use of second-order curl-conforming tetrahedral elements as those of [2], [5]. For the solution of the FEM system a frontal solver from [6] is used. The preprocess (geometry definition, mesh generation, etc) and postprocess (field visualization, etc) have been made with GiD [7].

The plane is taken into account in the analysis by means of PO where PO current density \vec{J}_{PO} is obtained from the incident magnetic field on the plane. The PO plane is discretized with regular quadrilateral elements of side equal to 0.1λ . Inside each element the current is approximated by linear functions which preserve its normal continuity between adjacent elements. Fig. 4 shows $|\vec{J}_{\text{PO}}^{(i)}|$ for the first six iterations of the hybrid method when a $\hat{\theta}$ polarized plane wave is illuminating the plane (of $W = 2\lambda$) from a normal direction. It may be observed how the current is modified. Initial current, $|\vec{J}_{\text{PO}}^{(0)}|$, is constant and its value is $2/\eta_0$. $|\vec{J}_{\text{PO}}^{(i+1)} - \vec{J}_{\text{PO}}^{(i)}|$ diminishes with each iteration and $|\vec{J}_{\text{PO}}|$ converges toward a function almost constant on the illuminated surface with small amplitude in the shade region. Furthermore, a small orthogonal polarization component is generated. Fig. 5 shows the bistatic RCS results with W as parameter. As the total scattered field increases with W , the scattered field shown in the figure is

the total scattering fields minus the reflected field by the plane when there is no FEM object (the cube). The results for the latter are obtained by the iterative FEM solution using dyadic Green's function with Dirichlet (Neumann) condition for E-formulation (H-formulation). It may be observed how the results obtained with the hybrid method for the finite size plane converge, as the plane width W is getting larger, to those obtained with FEM and an infinite width plane. The results mentioned above have been obtained using the E-formulation. Analogous results have been obtained with the H-formulation. The number of iterations of the method is around 25 for $\|b_{\Psi}^{(i)} - b_{\Psi}^{(i-1)}\| < 10^{-4}$.

4 Conclusions

A hybrid methodology for the efficient analysis of scattering problems where small and complex features are combined with electrically large objects has been presented. It makes use of FEM for the regions with small and complex features and an HF technique for the analysis of the electrically large objects of the problem. Specifically, its 3D implementation with PO as the HF technique is presented and numerical results are shown to illustrate the main features of the approach.

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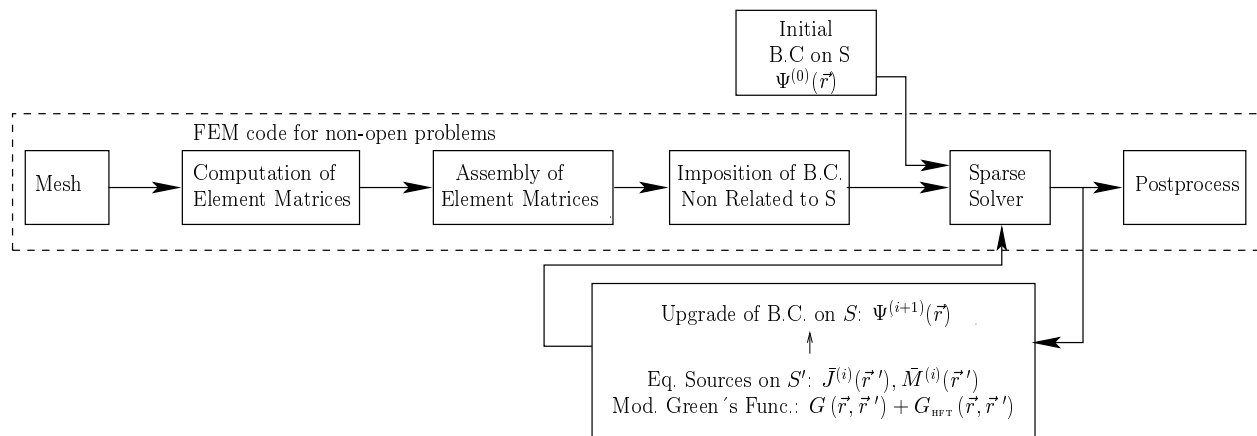


Fig. 2: Flow chart of the iterative hybrid FEM-HF Technique

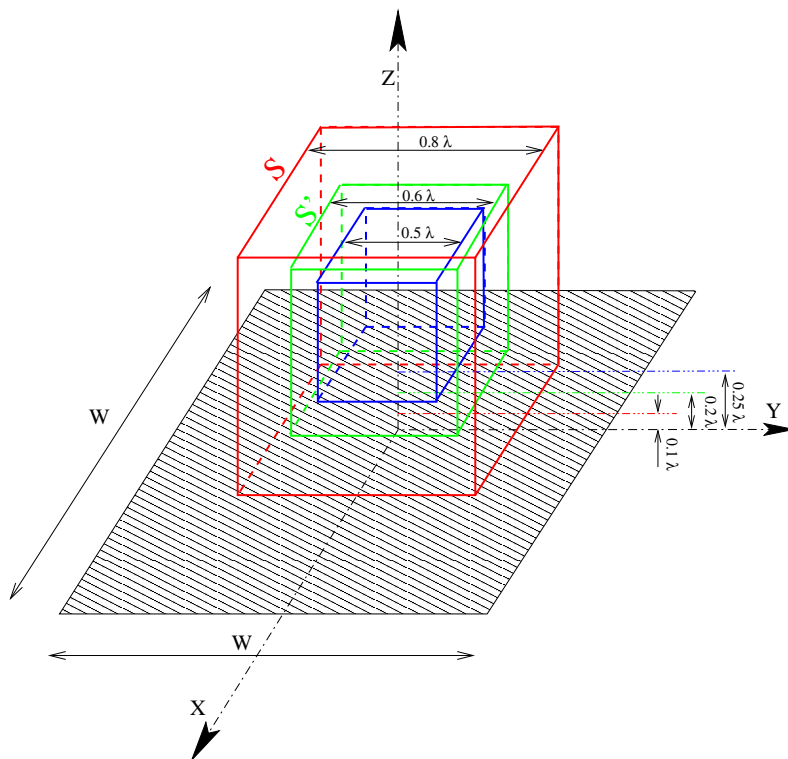


Fig. 3: Dielectric coated cube above a perfect conducting plane

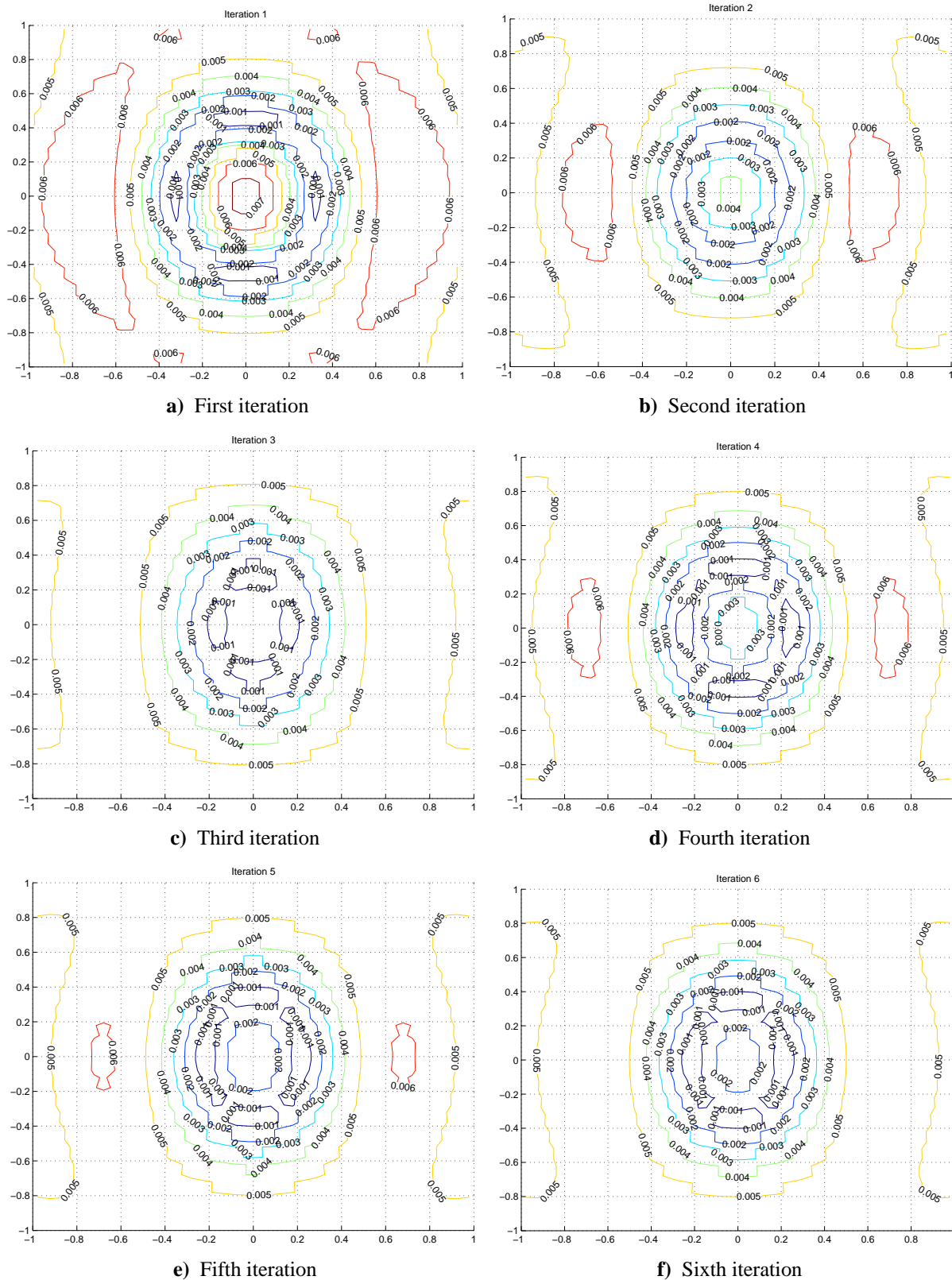


Fig. 4: $|\vec{J}_{PO}|$ on plane ($W = 2\lambda$) versus iteration number for problem of fig. 3

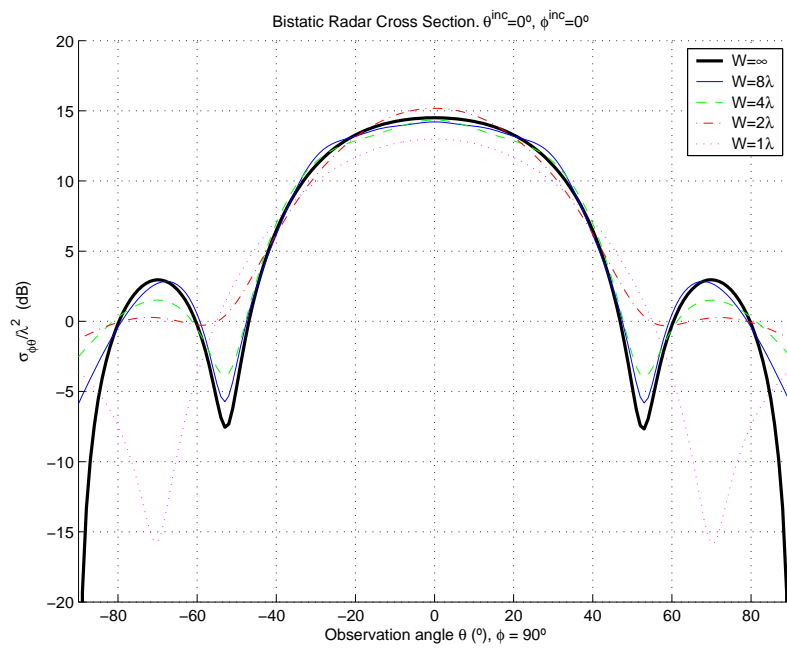
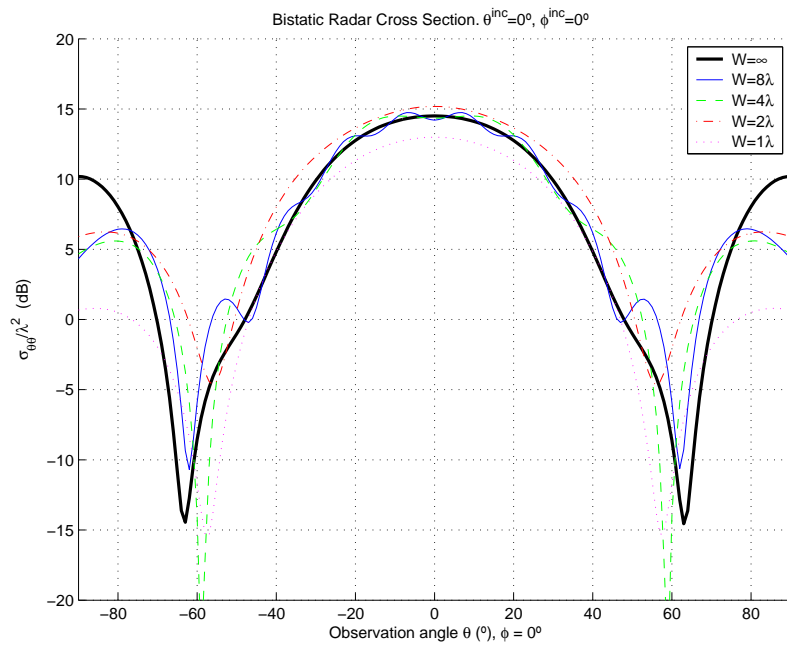


Fig. 5: Bistatic RCS for problem of fig. 3 for different W